strokes cursively, the two was two horizontal parallel bars and the eight two curved vertical lines, and some of the old forms of eight are practically the same as the two turned over 90°.

It is quite likely also that when forms of numerals are evidently tally marks, the ancient tribes would not stick to any particular arrangement, but form new ones provided they indicated numbers. This is the most reasonable explanation of the very evident tally-mark nature of the numerals in the Jaina manuscript. Ten is a nine with an extra stroke, and the eights are sevens with an extra stroke. The Jaina four, five, and six are also clearly derived from groups of marks. In course of time, by slurring, omission of strokes and adding embellishing flourishes, the Nepal and Bower manuscript forms arose. Indeed in the seven there is a perfect gradation of evolutionary forms to our present seven. In the four the resemblance is seen by making an assumption. In the five there is more evidence of an attempt to write cursively one of the X forms of the Chinese, but the six is not so evident without making two assumptions.

The supremacy of the Chinese numerals is explained by the fact that they were the first ideographs in the field. Egyptian pictographs evolved in the direction of representing sounds and, besides, their tally marks elsewhere were in groups of parallels, and not the fortunate Chinese groupings which lent themselves to change into ideographs. The invention of position value of course killed all the numerals above nine.

ON THE NUMBER OF EQUAL REGULAR SPHERICAL POLYGONS THAT CAN BE CONSTRUCTED SO AS TO COMPLETELY COVER A SPHERE.

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Let N=the number of equal polygons required; n=the number of sides in each, and k=the number of angles about each common vertex.

Then will $[\frac{360}{k}n-(n-2)180]N$=the area, in spherical degrees, of the sum of all the spherical polygons completely covering the sphere; but this area is also equal to 720.

Therefore, $[\frac{360}{k}n-(n-2)180]N=720$; whence, $N=\frac{4k}{2n-nk+2k}$.*

It remains to solve this equation for positive integers.

*This formula may be found on page 69, Vol. III, of Henrici and Treutline's *Geometry*, though developed by a different method, and having different considerations in view.

1. For $k=1$, $N=\frac{4}{n+2}$. Now, the only positive integral value n can have, to make N likewise integral, is 2; from which $N=1$. This is the case of a lune with its sides coincident and its angle 360°.

2. For $k=2$, $N=2$ for any value of n. This is the case of two hemispheres, each of which may be considered as bounded by any number of sides, each angle being an angle of 180°. In particular, n may equal 2, in which case each hemisphere is regarded as a lune.

3. For $k=3$, $N=\frac{12}{6-n}=3$, 4, 6, or 12, according as $n=2$, 3, 4, or 5, respectively. The figures are, in order, lunes, equilateral triangles, regular quadrangles, and regular pentagons.

4. For $k=4$, $N=\frac{8}{4-n}=4$ or 8, according as $n=2$ or 3. The corresponding figures are lunes or equilateral triangles.

5. For $k=5$, $N=\frac{20}{10-3n}=5$ or 20, according as $n=2$ or 3. The corresponding figures are lunes or equilateral triangles.

6. For $k=6$, $N=\frac{24}{12-4n}=6$, when $n=2$. [Lunes.] When $n=1$, $N=3$. But $k=6$, $n=1$, $N=3$ are incompatible with the nature of the problem.

7. For $k>6$, $n=2$, and $N=k$. From $N=\frac{4k}{2n-nk+2k}$, it is easily seen that, for $n=2$, $N=k$. This includes, as well, the six cases of lunes already considered. From the denominator $2n-nk+2k$ it is seen that $2n+2k$ must be greater than nk: $2n+2k>nk$.

Therefore, $n(k-2)<2k$, whence $n<\frac{2k}{k-2}=2+\frac{4}{k-2}$. Now it is clear that for $k>6$, $n<3$. That is, n must be 2 or 1. But the values of n have both been considered.

Table of Results:

$k=1$, $n=2$, $N=1$ . . .	Lune.
$k=2$, n, any value, $N=2$	Hemispheres.
$k=3$, $n=2$, $N=3$ . . .	Lunes.
$k=3$, $n=3$, $N=4$ . . .	Triangles...*A*.
$k=3$, $n=4$, $N=6$ . . .	Quadrangles...*B*.
$k=3$, $n=5$, $N=12$ . . .	Pentagons...*C*.
$k=4$, $n=2$, $N=4$ . . .	Lunes.
$k=4$, $n=3$, $N=8$ . . .	Triangles...*D*.
$k=5$, $n=2$, $N=5$ . . .	Lunes.
$k=5$, $n=3$, $N=20$ . . .	Triangles...*E*.
$k \overset{=}{>} 6$, $n=2$, $N=k$ . . .	Lunes.

From A, B, C, D, and E, we may easily pass to the proof that there are five and only five regular curvex polyhedrons.